

New Theories in Six Dimensions and Matrix Description of M-theory on T^5 and T^5/Z_2

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We present four infinite series of new quantum theories with super-Poincare symmetry in six dimensions, which are not local quantum field theories. They have string like excitations but the string coupling is of order one. Compactifying these theories on T^5 we find a Matrix theory description of M theory on T^5 and on T^5/\mathbb{Z}_2 , which is well defined and is manifestly U-duality invariant.

1. Introduction

The Matrix theory [1] description of compactification of M theory on T^3 involves a supersymmetric gauge theory in 3+1 dimensions [1-4]. Compactification on higher dimensional tori seems to involve higher dimensional gauge theories which are not renormalizable. Therefore, this prescription must be modified. Indeed, in [5,6] it was suggested to define the compactification on T^4 in terms of a six dimensional field theory with (2,0) supersymmetry. This field theory is at a fixed point of the renormalization group [7-10]. Trying to extend this discussion to compactification on T^5 has led the authors of [6] to conjecture the existence of a new theory (parametrized by an integer N) with six dimensional super-Poincare symmetry, which has string like excitations and T-duality. Having string like excitations does not necessarily mean that the theory is not a local quantum field theory. For example, the (2,0) theory has string like excitations, which become tensionless at the critical point, but the theory there appears to be a local quantum field theory. However, the existence of T-duality in our theory shows that the underlying geometry of the base on which the theory “lives” is ambiguous. Therefore, the theory does not have a standard energy momentum tensor, and it is not a local quantum field theory.

One of the purposes of this paper is to give an existence proof for this theory, and to explore some of its properties. Our analysis will lead us to find several such theories.

Our strategy will be to assume that M theory exists, and to study various 5-branes in the theory. We will focus on various string theory limits of M theory, and will explore the limits as the string coupling, g_s , goes to zero. In these limits, the modes in the bulk of space-time decouple. However, the modes on the branes remain interacting. The 5-branes are singular objects, and therefore even in these limits the theory on the 5-branes is still non-trivial.

We find four infinite classes (labeled by the number of parallel 5-branes, N) of new theories. Two of them are obtained from the NS5-branes of the type IIA and the type IIB string theories. The other two are found on 5-branes (instantons) of the $Spin(32)$ and the $E_8 \times E_8$ heterotic string theories.

Upon compactification on tori these theories inherit the T-duality of the underlying string theory. Two facts are crucial in establishing that T-duality acts on these theories. First, $g_s = 0$ is a fixed point of T-duality. Second, wrapped NS5-branes remain wrapped NS5-branes after T-duality. In this respect they differ from D-branes [11]. If these had been ordinary local quantum field theories, the base on which the fields propagate would have been unambiguous. Its geometry would have been related to the existence of a well defined energy momentum tensor. Here, the existence of T-duality makes the base ambiguous, and therefore these theories do not have a unique energy momentum tensor. Instead, there are

several different operators which can be interpreted as the energy momentum tensor. In different limits of the parameters different operators are naturally identified as the energy momentum tensor. (Note that this is unlike the situation in theories of gravity or in topological field theories, where there is no energy momentum tensor at all.) We conclude that our new theories are not local quantum field theories.

We should comment that somewhat related ideas were discussed in the context of black holes in [12]. The precise relation between this work and our proposal is not completely clear to us.

In section 2 we present two new classes of theories with 16 supercharges in six dimensions. One of them has (1,1) supersymmetry and its low energy behavior is a $U(N)$ gauge theory with 16 supercharges. The other has (2,0) supersymmetry, and its low energy behavior is an interacting field theory. They are continuously connected upon compactification on a circle. Compactifying this theory on T^5 we find a well defined and manifestly U-duality invariant description of M theory on T^5 .

In section 3 we present two new classes of theories with (1,0) supersymmetry in six dimensions. One of them has a $Spin(32)$ global symmetry. At low energies it becomes an $SP(N)$ gauge theory with 16 hypermultiplets in the fundamental representation and one hypermultiplet in the antisymmetric tensor representation. The other has $E_8 \times E_8$ global symmetry and is an interacting field theory at low energies. They are continuously connected upon compactification on a circle. Compactifying this theory on T^5 we find a well defined and manifestly U-duality invariant description of M theory on T^5/\mathbb{Z}_2 .

In section 4 we mention some extensions of our work and suggest the existence of other new theories.

2. New Theories with 16 supercharges

2.1. The Theories

Here we consider the theory of solitonic 5-branes in the two type II theories.

We start with N NS5-branes in the type IIB theory. Using S-duality we can relate this to N D5-branes. These have a $U(N)$ gauge symmetry [13] and gauge coupling

$$\frac{1}{g_D^2} = \frac{M_s^2}{g_s}, \quad (2.1)$$

where M_s is the string scale. In the weak coupling limit the gauge theory becomes weakly coupled. This means that the gauge theory on the D5-branes breaks down at energies of

order $M_s/\sqrt{g_s}$ and needs new dynamics at that scale. Using S-duality, the gauge coupling in the $U(N)$ gauge theory on the NS5-branes is

$$\frac{1}{g_{NS}^2} = M_s^2. \quad (2.2)$$

We see that even when $g_s = 0$ the gauge coupling does not vanish. The gauge theory on the NS5-branes breaks down at energies of order M_s and needs new degrees of freedom there. Since the underlying string theory makes sense, we expect that the theory of the NS5-branes also makes sense. It includes the necessary degrees of freedom at energies of order M_s to yield a consistent theory. It is crucial to stress that the theory we are left with is not the full underlying string theory. The latter has vanishing coupling constant and most of its modes decouple.

We do not have a complete description of the theory at energies of order M_s . However, using the low energy gauge theory we can detect that it includes strings whose tension is M_s^2 . The gauge theory has instantons which are strings in six dimensions. Their tension is $\frac{1}{g_{NS}^2} = M_s^2$. They can be interpreted as fundamental strings within the NS5-branes. Since $g_s = 0$, they cannot leave the branes. Similar instantons in D5-branes were studied in [14]. The detailed properties of these strings depend on the details of the theory which we do not know.

We can repeat this analysis for N NS5-branes in the IIA theory. Here we find at low energies a six dimensional theory with (2,0) supersymmetry. The moduli space of vacua is

$$\mathcal{M} = \frac{(\mathbb{R}^4 \times S^1)^N}{\mathbf{S}_N}. \quad (2.3)$$

The circle S^1 originates from the circle which relates M theory to the IIA theory. As in [10], we take the moduli, Φ , to be fields of dimension 2. In this normalization their kinetic terms have no dimensionful coefficient and the circumference of the S^1 factor is M_s^2 . At the singularities of the moduli space (2.3) we find the interacting (2,0) theory associated with the group $U(N)$ (for a review, see [10]). At the vicinity of these singularities the theory includes strings, whose tension vanishes at the singularities. Clearly, the full theory is not just this field theory. At energy of order M_s new degrees of freedom become important. Some of them can be identified from an M theory point of view. M theory membranes which wind around the circle and end on the 5-branes appear as strings whose tension is M_s^2 . As before, they can be identified as the fundamental strings which cannot leave the 5-branes because $g_s = 0$.

Consider now the compactification of the underlying type II string theory on a circle and wrap the NS5-branes on this circle. T-duality on this circle is not a symmetry. It relates

a compactification of the IIA theory on a circle with radius Σ^A with a compactification of the IIB theory on a circle of radius $\Sigma^B = \frac{1}{M_s^2 \Sigma^A}$. The theory in the non-compact directions is a five dimensional $U(N)$ gauge theory with coupling constant

$$\frac{1}{g_5^2} = \Sigma^B M_s^2 = \frac{1}{\Sigma^A}. \quad (2.4)$$

The moduli space of vacua is as in (2.3). From the IIA point of view we define the scalars $\phi = \Sigma^A \Phi$. From the IIB point of view four of the scalars are as in the higher dimensional theory and the fifth originates from the gauge fields. Its periodicity is $\frac{1}{\Sigma^B} = \Sigma^A M_s^2$.

It is also illuminating to compare some of the different BPS states in the five dimensional theory. For example, the “W-bosons,” which are obvious in the low energy IIB theory, arise as strings winding around Σ^A in the IIA theory. The momentum modes of the IIA theory with masses $\frac{n}{\Sigma^A}$ correspond to strings with tension M_s^2 (instantons) winding n times around Σ^B . Similarly, momentum modes of the IIB theory with masses $\frac{n}{\Sigma^B}$ correspond to strings with tension M_s^2 winding n times around Σ^A . This is the standard exchange of momentum and winding modes in T-duality done here for the modes on the 5-branes. It is an amusing exercise to compare also the strings in the five dimensional gauge theory.

We can continue further to compactify these theories on T^5 . The data of the compactification are the metric and the B field on the T^5 (the RR fields of the string theory decouple at $g_s = 0$). The B field couples to the strings in these theories. In terms of the gauge theory of the six dimensional IIB theory these strings are instantons and therefore the coupling is¹ $\int B \wedge \text{Tr } F \wedge F$. The total number of parameters in the compactification is 25 and they parametrize

$$SO(5, 5, \mathbb{Z}) \backslash SO(5, 5) / (SO(5) \times SO(5)), \quad (2.5)$$

which is the standard Narain space inherited from the underlying string theory. The full duality group $SO(5, 5, \mathbb{Z})$ follows as in string theory.

2.2. Some Excitations

Some of the excitations of these theories will be useful below. Consider a bound state of N NS5-branes and n Dp -branes (p even for IIA and p odd for IIB). For simplicity we

¹ We thank E. Witten for a useful discussion on this point.

consider a compactification on a torus with right angles and no B field. The energy of such a state in the $g_s = 0$ limit is

$$\begin{aligned}
E &= \lim_{g_s \rightarrow 0} \sqrt{\left(\frac{N\Sigma_1\Sigma_2\Sigma_3\Sigma_4\Sigma_5 M_s^6}{g_s^2}\right)^2 + \left(\frac{n\Sigma_{i_1}\dots\Sigma_{i_p} M_s^{p+1}}{g_s}\right)^2} - \frac{N\Sigma_1\Sigma_2\Sigma_3\Sigma_4\Sigma_5 M_s^6}{g_s^2} \\
&= \frac{(n\Sigma_{i_1}\dots\Sigma_{i_p})^2}{2NM_s^{4-2p}\Sigma_1\Sigma_2\Sigma_3\Sigma_4\Sigma_5},
\end{aligned} \tag{2.6}$$

where the factors of g_s are determined by the tensions of the objects – $\frac{1}{g_s^2}$ for the NS5-branes and $\frac{1}{g_s}$ for the D5-branes and the Σ_i 's are Σ_i^A or Σ_i^B depending on the description we use (NS5-branes in IIA or in IIB). Note that these states have finite energy for $g_s = 0$. Clearly, the T-duality group combines the wrapped even D-branes of the IIA theory (the even homologies) to a **16** of $SO(5, 5)$ and the wrapped odd D-branes to **16'** of $SO(5, 5)$.

The states with energies (2.6) can be identified in terms of excitations of our two low energy theories. They are characterized by carrying fluxes of the $U(1)$ part of the low energy gauge theories. Near the limit where the description as NS5-branes in IIA is appropriate all the fluxes are visible as in [6]. To keep the notation uniform we dualize the compact scalar to a four form gauge field, $A^{(4)}$, with a five form field strength, $F^{(5)}$, whose kinetic term is multiplied by $\frac{1}{M_s^4}$. This field has one magnetic flux and five electric fluxes. The energies of states carrying n units of these fluxes are easily computed using the free Lagrangian

$$\begin{aligned}
E^M &= \frac{n^2}{2NM_s^4\Sigma_1^A\Sigma_2^A\Sigma_3^A\Sigma_4^A\Sigma_5^A} \\
E_i^E &= \frac{M_s^4\Sigma_1^A\Sigma_2^A\Sigma_3^A\Sigma_4^A\Sigma_5^A n^2}{2N(\Sigma_i^A)^2},
\end{aligned} \tag{2.7}$$

where we have set the normalization (factor of 2) to agree with (2.6). In addition, there are ten fluxes of the self-dual H field. The energies of the corresponding states are

$$E_{ij}^H = \frac{(n\Sigma_i^A\Sigma_j^A)^2}{2N\Sigma_1^A\Sigma_2^A\Sigma_3^A\Sigma_4^A\Sigma_5^A} \tag{2.8}$$

where again, we have set the normalization to agree with (2.6). These can be identified with the 16 states in (2.6) corresponding to bound states of N NS5-branes with n D0-branes (E^M), n D2-branes (E_{ij}^H), or n D4-branes (E_i^E).

Near the region of the parameters where the light degrees of freedom are those of the (1,1) super-Yang-Mills theory the states can be identified as in [4]. Here we find 15 fluxes:

five electric and ten magnetic fluxes. States with n units of flux have energies

$$\begin{aligned} E_i^E &= \frac{(n\Sigma_i^B)^2}{2NM_s^2\Sigma_1^B\Sigma_2^B\Sigma_3^B\Sigma_4^B\Sigma_5^B} \\ E_{ij}^M &= \frac{M_s^2\Sigma_1^B\Sigma_2^B\Sigma_3^B\Sigma_4^B\Sigma_5^B n^2}{2N(\Sigma_i^B\Sigma_j^B)^2}, \end{aligned} \quad (2.9)$$

where again we have set the normalization to agree with (2.6). The bound states of N NS5-branes and n D1-branes correspond to the states with electric flux (E_i^E) while the bound states with n D3-branes correspond to the states with magnetic flux (E_{ij}^M). We can also use the T-duality transformation between the two descriptions, $\Sigma_{1,2,3,4}^B = \Sigma_{1,2,3,4}^A$, $\Sigma_5^B = \frac{1}{M_s^2\Sigma_5^A}$ to match with the states of (2.7) and (2.8). The remaining 16'th “missing” state has energy

$$E = \frac{n^2}{2N} M_s^6 \Sigma_1^B \Sigma_2^B \Sigma_3^B \Sigma_4^B \Sigma_5^B. \quad (2.10)$$

It corresponds to a bound state of N NS5-branes and n D5-branes (2.6). It has finite energy density, $n^2 M_s^6 / 2N$. Therefore, it corresponds to another sector of the Hilbert space of the infinite volume theory. This explains why it cannot be identified as an excitation in the low energy theory around the ordinary vacuum – in the super-Yang-Mills theory.

2.3. Matrix Theories of M Theory

We can now use these theories as Matrix theories of M theory as suggested in [6]. Consider M theory compactified on T^5 with sizes L_i ($i = 1, \dots, 5$). For simplicity we take all the angles to be right angles and we set the three form field to zero. To describe this theory we take N NS5-branes in IIB with scale M_s on T^5 with sizes Σ_i^B such that

$$\begin{aligned} \Sigma_i^B &= \frac{l_p^3}{RL_i} \\ M_s^2 &= \frac{R^2 L_1 L_2 L_3 L_4 L_5}{l_p^9}, \end{aligned} \quad (2.11)$$

where R is the length of the longitudinal direction and l_p is the eleventh dimensional Planck length. Alternatively, we can use N NS5-branes of the IIA theory with

$$\begin{aligned} \Sigma_{1,2,3,4}^A &= \frac{l_p^3}{RL_{1,2,3,4}} \\ \Sigma_5^A &= \frac{l_p^6}{RL_1 L_2 L_3 L_4} \\ M_s^2 &= \frac{R^2 L_1 L_2 L_3 L_4 L_5}{l_p^9}. \end{aligned} \quad (2.12)$$

In some range of parameters the first description becomes approximately a six dimensional gauge theory which coincides with the prescription of [4]. It is crucial to stress that this gauge theory is only an approximate description in some range of parameters; the complete theory is that of the NS5-branes. The second description with the parameters in (2.12) matches that of [6].

As explained in [6], already for the case of compactification on T^4 , there is no unique way to extract the space-time geometry from these theories. The point is that space-time is the moduli space of vacua of the theory. However, in quantum mechanics there is no moduli space of vacua – we have to integrate over all vacua. The closest to moduli space of vacua is a situation, as in the Born-Oppenheimer approximation, where some of the energy levels are much lower than others and almost form a continuum. Then we can describe them as degrees of freedom moving on some space – “the moduli space of vacua” – which is identified as the space-time of M theory. Already for the case of M theory on T^4 , this procedure was shown to be ambiguous; for different values of the space-time moduli (parameters in the quantum mechanics) we find different natural space-time interpretations [6]. Here, because of the T-duality, we see a phenomenon which is more subtle. Even the underlying base space on which the new theory propagates is ambiguous.

In addition to the 16 supercharges Q of our new theories there are also 16 non-linearly realized supercharges \tilde{Q} , which together give the 32 supercharges of M theory. This split between them is standard in the light-cone frame. As in [15], we write the supersymmetry algebra in the light-cone frame including the central charges. We set to zero all the central charges which are not scalars of the transverse space rotation group $SU(2) \times SU(2)$. This symmetry appears as an R-symmetry of our theories (the low energy theory on the NS5-brane in IIA has $SP(2)$ R-symmetry, which is broken at the scale M_s to an $SU(2) \times SU(2)$ subgroup). Together with the space rotation group the symmetry is $SU(2) \times SU(2) \times Spin(5)$. The supercharges transform under it as two copies of $(\mathbf{2}, \mathbf{1}, \mathbf{4}) \oplus (\mathbf{1}, \mathbf{2}, \mathbf{4})$. The supersymmetry algebra is

$$\begin{aligned}
\{Q_\alpha^i, Q_\beta^j\} &= 2H\epsilon_{\alpha\beta}J^{ij} + \epsilon_{\alpha\beta}\Gamma_I^{ij}Z^I \\
\{Q_{\dot{\alpha}}^i, Q_{\dot{\beta}}^j\} &= 2H\epsilon_{\dot{\alpha}\dot{\beta}}J^{ij} + \epsilon_{\dot{\alpha}\dot{\beta}}\Gamma_I^{ij}\tilde{Z}^I \\
\{Q_\alpha^i, \tilde{Q}_\beta^j\} &= \epsilon_{\alpha\beta}Z^{ij} \\
\{Q_{\dot{\alpha}}^i, \tilde{Q}_{\dot{\beta}}^j\} &= \epsilon_{\dot{\alpha}\dot{\beta}}\tilde{Z}^{ij} \\
\{\tilde{Q}_\alpha^i, \tilde{Q}_\beta^j\} &= 2P^+\epsilon_{\alpha\beta}J^{ij} \\
\{\tilde{Q}_{\dot{\alpha}}^i, \tilde{Q}_{\dot{\beta}}^j\} &= 2P^+\epsilon_{\dot{\alpha}\dot{\beta}}J^{ij}
\end{aligned} \tag{2.13}$$

(all other anticommutators vanish), where $\alpha, \beta = 1, 2$ and $\dot{\alpha}, \dot{\beta} = 1, 2$ label the spinors of the two $SU(2)$ factors, and $i, j = 1, \dots, 4$, $I = 1, \dots, 5$ and J^{ij} are the spinor indices, the vector index and the invariant tensor of the $Spin(5)$ space rotation group.

The central charges $Z^I, \tilde{Z}^I, Z^{ij}, \tilde{Z}^{ij}$ in (2.13) can be interpreted both in our Matrix theory and in the space-time M theory. Inspection of (2.13) shows that if Z^I or \tilde{Z}^I are not zero, and we rescale them by a constant n , then the energy of the BPS states is proportional to n . Therefore, if many states which are labeled by n become light, we recognize the dispersion relation of relativistic particles (energy proportional to the momentum). We interpret these central charges as momenta in our theory. From the type II string point of view Z^I and \tilde{Z}^I are the left and right moving momenta of the string. In various limits of the Narain moduli space, some of the excitations carrying these quantum numbers become light and we can interpret them as ordinary momenta. Again, the fact that we have 10 such momenta rather than 5 shows that these are not theories on a well defined five torus.

The space-time interpretation of Z^I and \tilde{Z}^I is the winding numbers of strings stretched along the longitudinal directions. This interpretation can be derived using the full underlying Lorentz symmetry (rather than just the transverse part) as in [15]. Alternatively, this follows from the fact that their energy, which is interpreted as P^- in space-time, is proportional to their charge [15]. As a consequence of this interpretation it is clear that Z^I and \tilde{Z}^I should be proportional to the length of the longitudinal direction R . Indeed, we see in (2.11) and (2.12) that the lengths scale like $\frac{1}{R}$ and therefore the momenta scale like R .

It is nice to check this interpretation of longitudinal strings with the momenta of the underlying Matrix theory in simpler contexts. For compactifications on S^1 the momentum in the 1+1 dimensional gauge theory was interpreted in [16,17] as the winding number of longitudinal strings. In the context of compactifications on T^2 and the IIB string theory in ten dimensions [18,16] the momenta along the two space dimensions were identified as the winding numbers of longitudinal NS and D-strings [16]. A new element appears in compactifications on T^4 . Here, a 4-brane wrapping the T^4 can lead to a longitudinal string. Its winding number was identified in [19,15] as the instanton number of the 4 + 1 dimensional gauge theory. Precisely this object was identified in [5] as the momentum along the fifth direction in the matrix description of this theory.

We now turn to the interpretation of the central charges Z^{ij} and \tilde{Z}^{ij} in (2.13). From the space-time picture it is clear that for particles (0-branes) $Z^{ij} = \tilde{Z}^{ij}$ while for 4-branes in the transverse directions $Z^{ij} = -\tilde{Z}^{ij}$. The latter are rather singular objects and therefore we set $Z^{ij} = \tilde{Z}^{ij}$. Rescaling the values of the charges of a state by n , we see from (2.13) that the energy $H = P^-$ scales like $\frac{n^2}{P^+}$ which is consistent with the interpretation of

these objects as particles. They correspond to one state of a wrapped 5-brane, ten states of wrapped 2-branes and five momentum modes. From the Matrix point of view these charges are the fluxes discussed above. States with nonzero charges are the bound states of our system with D-branes.

3. New Theories with 8 supercharges

In this section we construct two more theories starting with the 5-branes of the two heterotic theories at zero coupling. These two theories have (1,0) super-Poincare symmetry in six dimensions. Most of the conceptual issues are similar to the discussion in the type II theory and will not be repeated here.

One theory, based on N $Spin(32)$ instantons has as its low energy theory an $SP(N)$ gauge theory with hypermultiplets in the antisymmetric tensor of $SP(N)$ and 16 fundamentals [20]. The global symmetry of this theory is $Spin(32)$. Its gauge coupling is $\frac{1}{g^2} = M_s^2$. Again, it is important that g does not vanish for $g_s = 0$. The theory includes string like excitations, which can be interpreted as $SP(N)$ instantons. Their tension is $\frac{1}{g^2} = M_s^2$ and they can be interpreted as the fundamental heterotic strings [14]. Their detailed properties depend on the structure of the theory at energies of order M_s . Note that unlike [20], we propose a complete theory and not just an effective description at low energies.

The other new theory is based on instantons in the $E_8 \times E_8$ heterotic string. Here the low energy theory is an interacting quantum field theory, which has string like excitations [21]. For N such instantons the moduli space of the full theory is

$$\frac{(\mathbb{R}^4 \times (S^1/\mathbb{Z}_2))^N}{\mathbf{S}_N}. \quad (3.1)$$

The scalars in this theory have dimension 2 such that their kinetic terms are dimensionless. The size of the S^1/\mathbb{Z}_2 factor is M_s^2 .

As in the type II theory, upon compactification on a circle these two six dimensional theories become the same. Compactification on T^5 depends on 105 parameters in

$$SO(21, 5, \mathbb{Z}) \backslash SO(21, 5) / (SO(21) \times SO(5)). \quad (3.2)$$

As in the type II theory, the T-duality is inherited from the heterotic string and implies that these are not local quantum field theories.

We now use these theories as Matrix theories for M theory. We should find a six dimensional theory with (2,0) space-time supersymmetry. This leads us to guess that the

answer is the compactification of M theory on T^5/\mathbb{Z}_2 , which is the same as the IIB theory on K3 [22]. As a first indication that this is the right answer we recognize the moduli space (3.2) as the moduli space of vacua of this space-time theory. The local structure of the moduli space is fully determined by supersymmetry and the lack of space-time anomalies (these determine the number 21). The fact that the global structure is correct is less trivial.

The identification can be made more precise by going to the limit, where we can use the $SP(N)$ gauge theory compactified on T^5 . There we can find the low energy modes, and identify the moduli space of the quantum mechanical system, which can be interpreted as space-time. As in [23], one branch of the moduli space is $(T^5/\mathbb{Z}_2)^N/\mathbf{S}_N$, which corresponds to N zero branes moving on T^5/\mathbb{Z}_2 . This fact completes the identification of this compactification.

We can repeat the analysis in the type II theory and find the 26 momenta in our new theories as central charges in the space-time supersymmetry algebra. They correspond to longitudinal strings. Unlike the type II theory, here there are no D-branes and therefore no fluxes which can appear as central charges for particles in space-time. This is consistent with the lack of one form gauge fields in space-time.

4. Extensions of this work

One natural extension of this work is to compactify these theories on other five dimensional manifolds. For example, we can compactify our type II theories on five manifolds, which break half the supersymmetries like $K3 \times S^1$. This will extend the work of [24] and will give a description of M theory on $K3 \times S^1$ (note that these are different $K3$'s). Here the moduli space of space-time vacua is $SO(20, 4, \mathbb{Z}) \backslash SO(20, 4) / (SO(20) \times SO(4))$. It appears as the moduli space of parameters labeling the compactification of our new theories. As in the compactification on T^5 , this is not just the geometric moduli space but includes the B field and the stringy identifications (mirror symmetry).

We can also try to compactify the heterotic theories on five manifolds, which break half of the supersymmetries. This should yield a Matrix description of compactifications to six dimensions with (1,0) supersymmetry.

In all the examples we studied the NS5-branes were localized in \mathbb{R}^4 . This space can be replaced with any consistent string background. For example, we can compactify some of it. If we want to have an arbitrary number of NS5-branes, we should keep at least 3 non-compact directions. This leads us to study any of the 5-branes localized at points in $\mathbb{R}^3 \times S^1$, thus finding new non-trivial theories. It is possible that these theories give the Matrix model description of compactifications to lower dimensions.

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